

## N<sup>3</sup>LO approximate results for top-quark differential cross sections and forward-backward asymmetry

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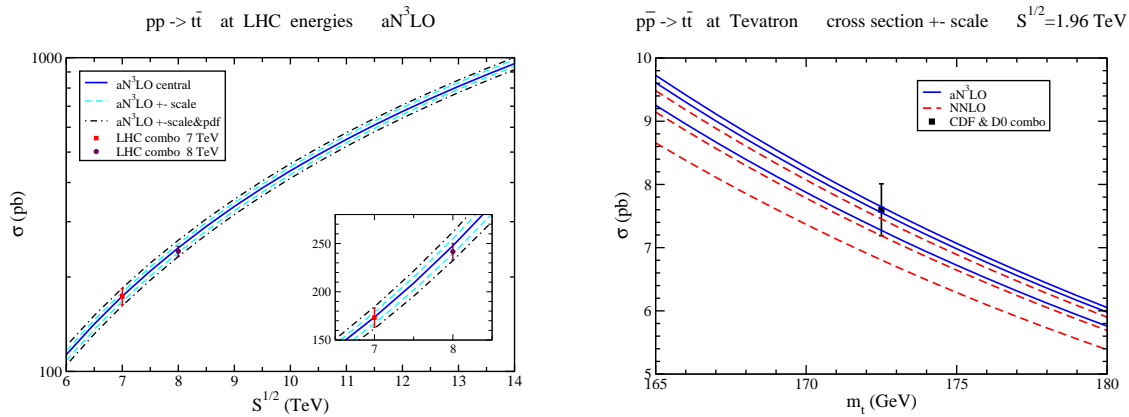
I present a calculation of approximate N<sup>3</sup>LO corrections from NNLL soft-gluon resummation for differential distributions in top-antitop pair production in hadronic collisions. Soft-gluon corrections are the dominant contribution to top-quark production and closely approximate exact results through NNLO. I show aN<sup>3</sup>LO results for the total  $t\bar{t}$  cross section, the top-quark  $p_T$  and rapidity distributions, and the top-quark forward-backward asymmetry. The higher-order corrections are significant and they reduce theoretical uncertainties.

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**Figure 1:** Total  $aN^3\text{LO}$  cross sections for  $t\bar{t}$  production at the LHC (left) and the Tevatron (right) and comparison with LHC [7, 8] and Tevatron [9] data.

## 1. Introduction

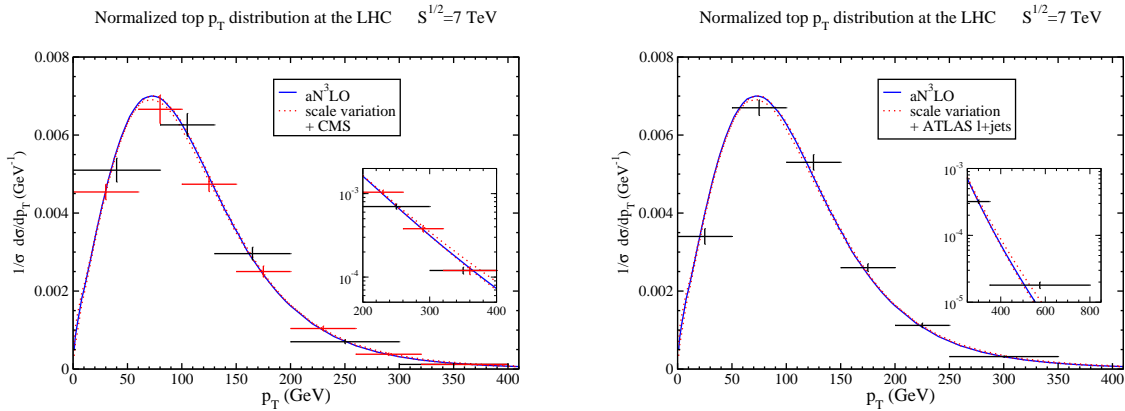
The calculation of higher-order corrections for  $t\bar{t}$  total cross sections, top-quark transverse momentum ( $p_T$ ) and rapidity distributions, and the top forward-backward asymmetry ( $A_{\text{FB}}$ ) is an important part of top-quark physics. QCD corrections are very significant for top-antitop pair production. Soft-gluon corrections, calculated appropriately, are the dominant part of these corrections at LHC and Tevatron energies. The soft corrections are currently known through  $N^3\text{LO}$  [1–3].

The soft-gluon terms in the  $n$ th-order perturbative corrections involve  $[\ln^k(s_4/m_t^2)]/s_4$  with  $k \leq 2n - 1$  and  $s_4$  the kinematical distance from partonic threshold. We resum these soft corrections at NNLL accuracy via factorization and renormalization-group evolution of soft-gluon functions [4]. The calculation is for the double-differential cross section using the standard moment-space resummation in perturbative QCD. The first  $N^3\text{LO}$  expansion was given in [5] with a complete formal expression given in [6]. Approximate  $N^3\text{LO}$  ( $aN^3\text{LO}$ ) total and differential cross sections from the expansion of the NNLL resummed expressions have been obtained most recently in [1, 2]. The latest  $aN^3\text{LO}$  results for the total cross section [1], top  $p_T$  and rapidity distributions [2], and the top forward-backward asymmetry  $A_{\text{FB}}$  [3], provide the best and state-of-the-art theoretical predictions.

It has been known for some time that the partonic threshold approximation in our formalism works very well for LHC and Tevatron energies; the differences between approximate and exact cross sections at both NLO and NNLO are at the per mille level. This is also true for  $p_T$  and rapidity distributions and  $A_{\text{FB}}$ . The use of a fixed-order expansion removes the need for a prescription to deal with divergences and the unphysical effects of such prescriptions. The stability and robustness of the theoretical higher-order results in our resummation approach over the past two decades as well as the correct prediction of the size of the exact NNLO corrections validate our formalism.

## 2. Top-antitop pair total cross sections at the LHC and the Tevatron

In Fig. 1 we show the  $aN^3\text{LO}$  total  $t\bar{t}$  cross sections at LHC and Tevatron energies [1] and



**Figure 2:** Normalized  $aN^3\text{LO}$  top-quark  $p_T$  distributions at the 7 TeV LHC, and comparison with CMS data [11] in the dilepton (black) and lepton+jets (red) channels (left plot), and with ATLAS data [12] in the lepton+jets channel (right plot).

compare them with LHC combination data from the ATLAS and CMS collaborations at 7 TeV [7] and 8 TeV [8] energies, and Tevatron combination data from the CDF and D0 collaborations at 1.96 TeV energy [9]. We use MSTW2008 NNLO pdf [10] for all our predictions. The agreement of theoretical predictions with experimental data is excellent.

We also provide the  $aN^3\text{LO}$  total  $t\bar{t}$  cross sections with  $m_t = 173.3$  GeV below. At the Tevatron with 1.96 TeV energy the cross section is  $7.37^{+0.09+0.38}_{-0.27-0.28}$  pb; at the 7 TeV LHC it is  $174^{+5+9}_{-7-10}$  pb; at the 8 TeV LHC it is  $248^{+7+12}_{-8-13}$  pb; at the 13 TeV LHC it is  $810^{+24+30}_{-16-32}$  pb; and at the 14 TeV LHC it is  $957^{+28+34}_{-19-36}$  pb. The first uncertainty in the previous numbers is from scale variation over  $m_t/2 \leq \mu \leq 2m_t$  and the second is from the MSTW2008 pdf [10] at 90% C.L.

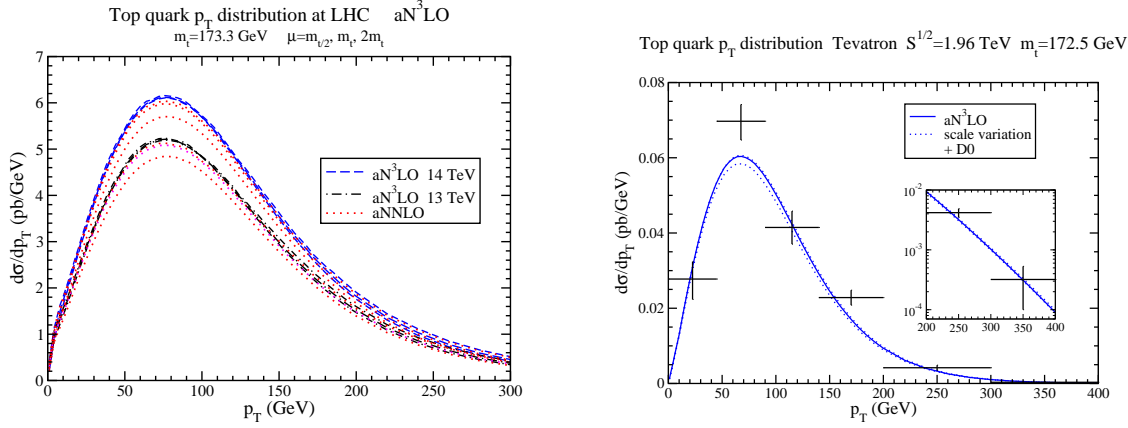
Fractional contributions to the perturbative series for the  $t\bar{t}$  cross section at the LHC converge well through  $N^3\text{LO}$ , which could potentially indicate that corrections beyond  $N^3\text{LO}$  are negligible [1]. For Tevatron energies the convergence is slower [1].

### 3. Top-quark $p_T$ and rapidity distributions at the LHC and the Tevatron

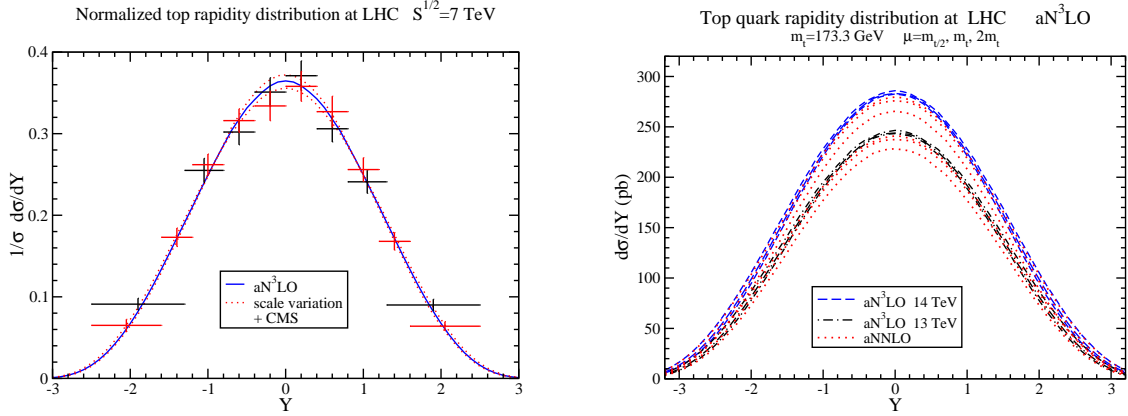
In Fig. 2 we show the normalized  $aN^3\text{LO}$  top-quark  $p_T$  distribution,  $(1/\sigma)d\sigma/dp_T$ , at 7 TeV LHC energy and compare with results from CMS in the dilepton and lepton+jets channels [11] and from ATLAS in the lepton+jets channel [12]. We find excellent agreement between the theoretical results and the 7 TeV LHC data. The theoretical predictions are also in excellent agreement with recent CMS top  $p_T$  data at 8 TeV in both channels [13].

In the left plot of Fig. 3 we show the  $aN^3\text{LO}$  top-quark  $p_T$  distributions [2],  $d\sigma/dp_T$ , at 13 and 14 TeV LHC energies. In the right plot of Fig. 3 we show the  $aN^3\text{LO}$  top-quark  $p_T$  distributions [2] at 1.96 TeV Tevatron energy and compare with D0 data [14], finding very good agreement.

We continue with the top-quark rapidity distribution at the LHC [2]. In the left plot of Fig. 4 we show the normalized  $aN^3\text{LO}$  top-quark rapidity distribution,  $(1/\sigma)d\sigma/dY$ , at 7 TeV LHC energy and compare with results from CMS in the dilepton and lepton+jets channels [11], finding excellent agreement between theory and data. The theoretical predictions at 8 TeV are also in



**Figure 3:** Top-quark aN<sup>3</sup>LO  $p_T$  distributions at the LHC (left) and at the Tevatron compared to D0 data [14] (right).



**Figure 4:** (Left) Top-quark aN<sup>3</sup>LO normalized rapidity distributions at the 7 TeV LHC and comparison with CMS data [11] in the dilepton (black) and lepton+jets (red) channels; (Right) Top-quark aN<sup>3</sup>LO rapidity distributions at 13 and 14 TeV LHC energies.

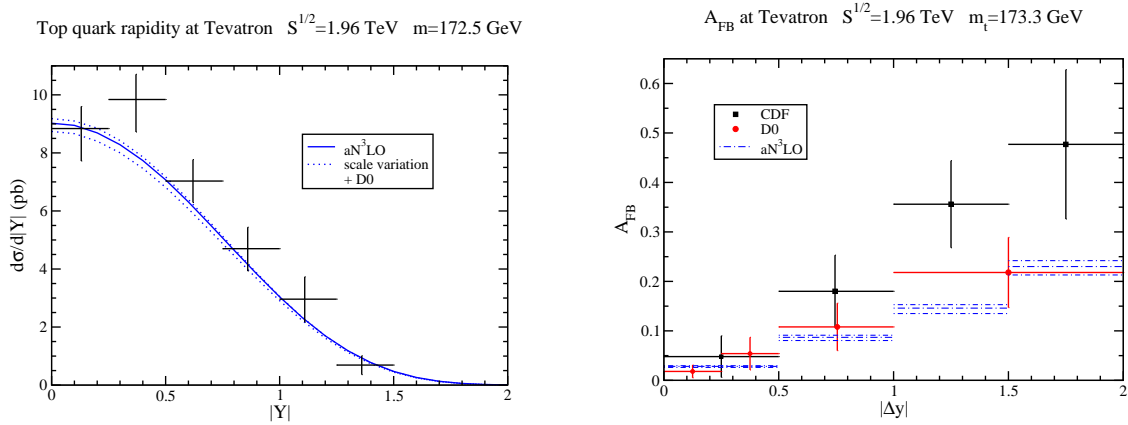
excellent agreement with recent CMS top rapidity data in both channels [13]. We also show the aN<sup>3</sup>LO top-quark rapidity distributions,  $d\sigma/dY$ , at 13 and 14 TeV LHC energies in the right plot of Fig. 4.

In the left plot of Fig. 5 we compare the aN<sup>3</sup>LO distribution of the absolute value of the top-quark rapidity,  $d\sigma/d|Y|$ , at the Tevatron with D0 data [14] and find very good agreement.

#### 4. Top-quark forward-backward asymmetry at the Tevatron

Finally, we discuss the top forward-backward asymmetry at the Tevatron

$$A_{\text{FB}} = \frac{\sigma(y_t > 0) - \sigma(y_t < 0)}{\sigma(y_t > 0) + \sigma(y_t < 0)}. \quad (4.1)$$



**Figure 5:** (Left) Top-quark aN<sup>3</sup>LO  $d\sigma/d|Y|$  distribution at the Tevatron compared with D0 data [14]; (Right) Top-quark aN<sup>3</sup>LO differential  $A_{FB}$  at the Tevatron compared with CDF [16] and D0 [17] data.

The above expression can be evaluated with numerator and denominator separately at fixed-order or it can be re-expanded in  $\alpha_s$  (see [3] for details through aN<sup>3</sup>LO). As was discussed in [3] the soft-gluon corrections are dominant and in our formalism they precisely predicted [15] the exact asymmetry at NNLO. The high-order perturbative corrections are large: the aN<sup>3</sup>LO/NNLO ratio is 1.08 without re-expansion in  $\alpha_s$ , or 1.05 with re-expansion in  $\alpha_s$ . Including electroweak corrections and the aN<sup>3</sup>LO QCD corrections we find an asymmetry of  $(10.0 \pm 0.6)\%$  in the  $t\bar{t}$  frame using re-expansion in  $\alpha_s$ .

The differential top forward-backward asymmetry is defined by

$$A_{FB}^{\text{bin}} = \frac{\sigma_{\text{bin}}^+(\Delta y) - \sigma_{\text{bin}}^-(\Delta y)}{\sigma_{\text{bin}}^+(\Delta y) + \sigma_{\text{bin}}^-(\Delta y)} \quad \text{with } \Delta y = y_t - y_{\bar{t}}.$$

In the right plot of Fig. 5 we plot the differential  $A_{FB}$  and compare with recent results from CDF [16] and D0 [17]. The agreement between theory and experiment is very good for both the total and the differential asymmetries.

## 5. Summary

The N<sup>3</sup>LO soft-gluon corrections for top-antitop pair production are significant and provide the best available theoretical predictions. Results have been presented for the total  $t\bar{t}$  cross sections, the top-quark  $p_T$  and rapidity distributions, and the top-quark forward-backward asymmetry. The corrections are large at LHC and Tevatron energies and they reduce the theoretical uncertainties from scale variation. There is excellent agreement between aN<sup>3</sup>LO theoretical predictions and LHC and Tevatron data.

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